# Towards Integrated Multidisciplinary Synthesis of Actively Controlled Fiber Composite Wings

E. Livne,* L. A. Schmit,† and P. P. Friedmann‡
*University of California, Los Angeles, Los Angeles, California 90024*

**The synthesis of actively controlled composite wings is formulated as a multidisciplinary optimization problem. A unique integration of analysis techniques spanning the disciplines of structures, aerodynamics, and controls is described. A rich variety of behavior constraints can be treated including stress, displacement, control surface travel and hinge moment, natural frequency, aeroservoelastic stability, gust response, and handling quality constraints, as well as performance measures in terms of drag/lift coefficients, drag polar shape, required load factor or roll rate, and wing mass. The design space includes a simultaneous treatment of structural, aerodynamic, and control system design variables. The paper sets the stage for multidisciplinary wing optimization by describing the capabilities and discussing the accuracy of the analysis and related behavior sensitivity analysis. Applicability of approximation concepts to the multidisciplinary optimization problem is examined by studying typical aeroservoelastic stability, gust response, and performance-related constraints. The computational efficiency of the combined analysis and sensitivity as well as the quality of key behavior constraint approximations indicate that single-level optimization of composite, actively controlled practical wings is within reach.**

## Nomenclature

| | |
|---|---|
| $a_0, a_1 \ldots a_n$ | = coefficients of the numerator polynomial in a sensor transfer function (see Fig. 3) |
| $[A],[B],[C],[D]$ | = Linear time invariant (LTI) state space equation matrices |
| $[\tilde{A}],[\tilde{B}]$ | = state space system matrices in standard form |
| $b_l$ | = aerodynamic lag terms |
| $b_l^G$ | = aerodynamic gust lag terms |
| $b_i$ | = control law transfer function numerator coefficients |
| $c_0, c_1 \ldots c_n$ | = coefficients of the denominator polynomial in a sensor transfer function (see Fig. 3) |
| $[C]$ | = damping matrix |
| $[\hat{C}]$ | = aerodynamically modified damping matrix in aeroservoelastic analysis |
| $d_i$ | = transfer function denominator coefficients |
| $e_0, e_1 \ldots e_n$ | = coefficients of the numerator polynomial in an actuator transfer function (see Fig. 3) |
| $[E],[F],[G],[H]$ | = aeroelastic system matrices |
| $f_0, f_1 \ldots f_n$ | = coefficients of the denominator polynomial in an actuator transfer function (see Fig. 3) |
| $F(X)$ | = objective function |
| $\{g\}$ | = vector of behavior constraint functions |
| $G_A$ | = aileron gain (see Fig. 7) |
| $[K]$ | = stiffness matrix |
| $[\bar{K}]$ | = aerodynamically modified stiffness matrix in maneuver load analysis |
| $[\hat{K}]$ | = aerodynamically modified stiffness matrix in aeroservoelastic analysis |
| $[M]$ | = mass matrix |
| $[\hat{M}]$ | = aerodynamically modified mass matrix in aeroservoelastic analysis |
| $MS$ | = mean square value of response to gust |
| $MS_0$ | = reference mean square value of gust response |
| $n_{ACT}$ | = number of actuator states |
| $n_{AER}$ | = number of aerodynamic added states |
| $n_{CO}$ | = number of control law states |
| $n_c$ | = number of control surface degrees of freedom (DOF) |
| $n_G$ | = number of gust filter states |
| $n_s$ | = number of structural DOF |
| $n_{SE}$ | = number of sensor states |
| $n_{SYS}$ | = number of aeroservoelastic states |
| $N_l$ | = number of Roger lag terms |
| $N_l^G$ | = number of Roger lag terms added for gust |
| $p$ | = any design variable |
| $\{P\}$ | = load vector |
| $[P_1]$-$[P_6]$ | = minimum state or Roger approximation matrices |
| $\{q\}$ | = generalized displacements |
| $\{q_c\}$ | = control surface deflection |
| $q_D$ | = dynamic pressure |
| $\{q(s)\}$ | = the vector of Laplace transformed generalized displacements |
| $\{Q_G(s)\}$ | = the Laplace transformed gust vector |
| $[Q(s)]$ | = generalized aerodynamic force matrix |
| $[Q_w]$ | = white noise intensity matrix |
| $\{r^A\}$ | = aerodynamic added states |
| $\{r^G\}$ | = aerodynamic gust added states |
| $s=\sigma+j\omega$ | = Laplace variable |
| $S$ | = reference area |
| $t_1$ | = skin thickness term |
| $\{u\}$ | = vector of control surface excitations [see Eq. (39)] |
| $U_\infty$ | = flight speed |
| $w$ | = white noise input |
| $w_G$ | = vertical gust velocity |
| $W_G(s)$ | = the Laplace transformed vertical gust velocity |

Presented as Paper 89-1268 at the 30th Structures, Structural Dynamics, and Materials Conference, Mobile, AL, April 3–5, 1989; received Aug. 9, 1989; revision received April 1, 1990; accepted for publication April 18, 1990. 

*Graduate Research Assistant. Student Member AIAA.
†Professor of Engineering and Applied Science. Fellow AIAA.
‡Professor and Chairman. Associate Fellow AIAA.

$[U],[V],[W]$ = aeroservoelastic system matrices
$\{x\}$ = state vector
$X$ = vector of design variables
$[X]$ = state covariance matrix
$y$ = output
$\delta_i$ = input command to an actuator
$[\Phi_0],[\Phi_1],[\Phi_2]$ = structural response transformation matrices
$\zeta$ = equivalent viscous damping
$\zeta_0$ = reference equivalent viscous damping

*Subscripts*
0 = reference value
ACT = actuator
CO = control law
$c$ = control surface
$G$ = gust
$s$ = structural DOF
SE = sensor

*Superscripts*
$G$ = gust
$L$ = lower bound
$U$ = upper bound

## Introduction

THE introduction of active control technology[1-6] and composite structural tailoring[7-11] to airplane wing design during the last 15 years requires a re-examination of the design practice followed in the past, which was based on a sequential, compartmented approach.

Using the sequential approach, undesirable interactions between disciplines, which were not taken into account properly during wing development, have resulted in expensive, sometimes lengthy, modifications and fixes.[12-16] At the same time there has been a growing recognition of the potential improvements possible when an integrated multidisciplinary design and synthesis approach is followed, in which all relevant disciplines are considered simultaneously.[17-19]

Three decades of extensive research and development have made optimization techniques widely accepted in every major discipline needed in wing design synthesis.[20] Structural synthesis has matured in the last decade and has already been used to size complex structures under various static and dynamic behavior constraints including flutter and static aeroelastic constraints.[21-38] Research carried out during the last 20 years on control system design, active flutter suppression, and gust alleviation has produced several alternative implementations of optimization to active control synthesis practices.[39-46] In the aerodynamic field, optimization has been used to synthesize wing camber, cross section, and planform to achieve desirable aerodynamic characteristics and performance.[47-58]

The growing confidence in modern analysis techniques and disciplinary optimization has led to a departure from conventional designs. It has become apparent that multidisciplinary interactions must be taken into account to prevent failure and to extract maximum benefits from the design freedom offered by a truly integrated approach. During the last decade, control augmented structural synthesis has emerged as an important research area.[59-66] Initial results, for wing design, have been recently reported emphasizing the multidisciplinary structural/aerodynamic synthesis of wings.[67-71] An initial study of the aeroservoelastic optimization problem, namely the simultaneous synthesis of wing structures and their active control systems, was also reported recently.[72] Methods for control law and control system performance sensitivity analysis with respect to structural and control system parameters have also been reported in recent years as a precursor to the application of multilevel decomposition techniques for design optimization.[73-75] An approach to the integrated handling qualities/aeroelastic stability design problem is given in Ref. 76. Parametric studies reported in Refs. 77 and 78 further highlight the importance of multidisciplinary design considerations for practical fighter wings. An integrated approach to the optimization of airplane configuration and control system was presented in Refs. 79 and 80 using simplified mathematical models. However, the application of modern optimization techniques to wing design involving a diverse mix of constraints based on analyses from several disciplines (structures, structural dynamics, aeroelasticity, aerodynamics, control, handling qualities) has not yet been treated in a comprehensive and realistic manner.

The purpose of this paper is to outline a unified framework for multidisciplinary wing synthesis. It contains a description of a set of techniques for analysis and behavior sensitivity analysis of actively controlled fiber composite wings, which will facilitate integrated design optimization. Several aspects of structural, aerodynamic, and control system modeling are discussed. Computational efficiency and accuracy of approximations for a rich variety of behavior constraints are examined. The paper lays the foundation for the application of approximation concepts and optimization techniques to practical control augmented aeroelastic wing design.

## The Complexity and Multidisciplinary Nature of Wing Design

The set of wing design descriptors, whose elements consist of preassigned parameters and design variables,[22] is shown in Fig. 1. Discussion is limited to wings operating in the sub-

| | STRUCTURES/ STRUCTURAL DYNAMICS | AERODYNAMICS | CONTROL |
|---|---|---|---|
| SIZING DESIGN DESCRIPTORS: (DESIGN VARIABLES OR PREASSIGNED PARAMETERS) | ELEMENT SIZE (AREA, THICKNESS) | CAMBER, LE/TE CONTROL SURFACE DEFLECTION | GAINS, TRANSFER FUNCTION COEFFICIENTS |
| CONFIGURATION DESIGN DESCRIPTORS: | PLANFORM SHAPE, (SWEEP, AR, TAPER RATIO) AIRFOIL CROSS SECTION, PLY ANGLE, RIB/SPAR LOCATIONS | PLANFORM SHAPE, (SWEEP, AR, TAPER RATIO) AIRFOIL CROSS SECTION | ORDER OF TRANSFER FUNCTIONS |
| TOPOLOGICAL DESIGN DESCRIPTORS: | NUMBER OF RIBS, SPARS, PLY ANGLES | NUMBER OF LE,TE CONTROL DEVICES | CONTROL SYSTEM STRUCTURE AND CONNECTIVITY |

**Fig. 1 Hierarchy of design descriptors in wing synthesis.**

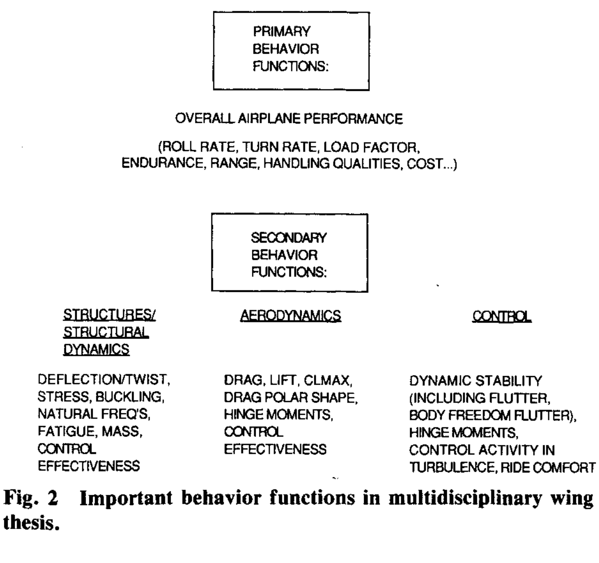


**Fig. 2 Important behavior functions in multidisciplinary wing synthesis.**

sonic to low-supersonic flight speeds, so that thermal effects can be neglected. A given set of design descriptors completely defines a particular wing design. Which of the descriptors will be preassigned parameters and which will be used as design variables depends on the level of application for optimization techniques in the hierarchy described in Ref. 22, namely, whether the design space includes sizing, configuration (geometry), or topological design variables. The set of behavior functions, from which constraints and objectives will be selected, can be divided into two categories (Fig. 2). Primary (system level) behavior functions are those performance measures that determine the overall quality and competitiveness of the wing. Secondary (subsystem level) behavior functions are the behavior functions that must be taken into account during the design to guarantee the prevention of failure in all possible failure modes and to introduce known constraints on subsystem performance. These are the means necessary to achieve the overall design goals and should ideally be "transparent" compared with real design objectives, although sometimes there can be strong correlation between a secondary behavior and a primary behavior function (e.g., mass and airplane performance).

The importance of multidisciplinary interactions in wing design is evident from Figs. 1 and 2. Structural topology, shape and sizing; control system topology, control law transfer function order, and gain values; as well as aerodynamic configuration layout, jig shape, and control surface deflections in maneuvers all interact to achieve desired wing performance while ensuring structural integrity, aeroservoelastic stability, ride comfort, and good handling qualities.[82-93]

The design synthesis problem can be cast in a mathematical programming form:

$$\min_{\{X\}} F(\{X\}) \tag{1}$$

$$s.t. \ \{g\} \leq \{0\}$$

$$\{X^L\} \leq \{X\} \leq \{X^U\}$$

To overcome the inherent complexity and to address the computationally intensive nature of this problem, two approaches have been suggested in the literature. The first approach is based on the application of multilevel decomposition techniques combined with existing tools for detailed analysis and sensitivity analysis for each of the disciplines.[94,95] The second approach seeks to gain some insight into the nature of the problem by using highly simplified mathematical models or simple airplane configurations for structural, aerodynamic, and control system analysis.[72,79,80]

## Modeling Considerations

### Structural and Aerodynamic Modeling

In Ref. 20, Ashley points out that a considerable part of the research done on wing optimization has been based on models that are "a long way from the complicated, built up lifting surfaces of real aircraft with their multiple design criteria and constraints." He warns that "very undesirable consequences can result from the omission or careless handling of constraints," and this warning is particularly relevant for multidisciplinary synthesis, where only limited experience exists to guide the designer, and intuition may sometimes be misleading.

The prevalent structural beam/aerodynamic strip models used for basic research in aeroelasticity are often inadequate when it comes to synthesizing real wings.[96-98] More realistic models are needed. At the same time, detailed finite element models and Computational Fluid Dynamics (CFD) aerodynamic techniques are still computationally too expensive to use within the inner loop of a multidisciplinary synthesis approach. Thus, it is necessary to bridge the gap between the highly idealized and the very detailed modeling alternatives by introducing balanced analysis and design optimization models that capture essential behavior characteristics, without making the integrated multidisciplinary design optimization task intractable.

The integrated optimum design capability outlined here is based on modeling and analysis techniques for the required disciplines, which are consistent with each other in terms of accuracy and efficiency and, thus, lead to a balanced treatment of practical wings. In the structures area, a rather general equivalent plate analysis,[98] which builds on the basic ideas underlying the Aerodynamic Tailoring and Structural Optimization (TSO) computer code[7,28,29] and incorporates additional recent developments due to Giles,[99,100] is used. The equivalent plate approach for structural modeling of low aspect ratio wings has been known for many years. It was Giles, however, who showed that, using present day computers, a single high-order power series can be used for approximating displacements over wing planforms made of several trapezoidal segments to obtain accurate stress as well as displacement information. Stresses in spar and rib caps can be calculated in addition to composite skin stresses. Configurations made of several plate segments attached to each other via springs accounting for attachment stiffness and actuator stiffness can be analyzed to simulate wing/control surface configurations. The simplicity of manipulating simple power series leads to analytic rather than numerical integration for the mass and stiffness expressions. With the careful organization of computer storage space and ordering of calculations, major savings in computation times and core storage requirements can be achieved.

In the work described here, the equivalent plate structural analysis documented in Ref. 98 is integrated with the Piecewise Continuous Kernel Function Method (PCKFM) developed by Nissim and Lottati for lifting surface unsteady aerodynamics.[101-104] Lifting surface unsteady aerodynamics[105,106] has served as the basic aerodynamic modeling tool for the flutter analysis of airplanes since the 1960s. The PCKFM combines the power of the doublet lattice method in dealing with pressure singularities with the accuracy and speed of the kernel function method. Extensive numerical experimentation has demonstrated[101] that PCKFM is accurate and converges rapidly. For configurations involving control surfaces, it can take narrow gaps into account, is faster than lattice methods, and is more accurate in the calculation of control surface hinge moments. Thus, it is particularly suitable for calculating the generalized unsteady air loads (on lifting surfaces made up of wing and control surface elements) that are needed for active flutter suppression and gust alleviation studies.

The combination of modern equivalent plate structural modeling and PCKFM lifting surface aerodynamics is thought to be adequate for the preliminary design of airplane wings and for the exploratory venture into multidisciplinary practical wing synthesis. In addition to a reliable prediction of flutter results and static aeroelastic effects, useful hinge moment[106] and induced drag predictions[107,108] can be expected for subsonic and supersonic small angle-of-attack flight. The analysis is adequate for addressing flight stability and control problems of the elastic airplane.[109] Its aerodynamic predictions might be improved by using correction factor techniques if any measured data are available.

### Finite-Dimensional State Space Modeling of Unsteady Aerodynamics

It is suggested in the literature that aeroservoelastic stability analysis can be successfully based on the *p-k* method using generalized aerodynamic force matrices computed for simple harmonic motion.[37,110] However, when the optimization of the design for aeroservoelastic stability is addressed and modern control techniques are to be implemented, it is necessary to cast the aeroelastic equations of motion in Linear Time Invariant (LTI) state space form. It then follows that some approximation of the unsteady aerodynamic loads in terms of rational functions of the Laplace variable is needed.

The method of Roger[111] has been widely used for finite-dimensional unsteady aerodynamic loads representation during the past decade. A series of aerodynamic stiffness, damping, apparent mass, and several lag terms is used to approximate elements of the generalized aerodynamic loads and gust force matrices over a range of reduced frequencies. A least-squares fitting procedure is carried out for each element of the matrices separately.

The minimum state method, developed by Karpel[112] and recently studied in Ref. 113, is found to be attractive because it has the potential for generating accurate approximations to unsteady generalized aerodynamic forces, while adding only a small number of states to the mathematical model of the aeroservoelastic system. It is based on an iterative fitting process in which all terms of the generalized aerodynamic load and gust force matrices are considered simultaneously. In comparison with other finite state modeling techniques, the number of states needed in the minimum state method appears to be smaller for the same overall accuracy of approximation.[113] This leads to a state space model of lower order, thus reducing core requirements and computation time.

Considering the accumulated experience and fast generation of Roger approximations (resulting, however, in higher-order mathematical models of the aeroservoelastic system) vs the smaller-order mathematical models possible with the minimum state approach (with a relative lack of experience and time-consuming approximant generation as potential handicaps), it was decided to include both methods in the present capability as available alternatives.

### Control System Modeling

The integrated aeroservoelastic system is modeled as an LTI system. Since the number of sensors and control surfaces is small in real airplanes, the complex, high-order laws generated by some multivariable control system design techniques are avoided at this stage. A schematic block diagram of the actively controlled aeroservoelastic system is shown in Fig. 3. Airplane motions (acceleration and angular rates) are measured by a set of sensors placed on the structure. The resulting signals are used as inputs to the control law block which commands control surface actuators. The control surface motions guarantee stability and desirable dynamic response of the complete system.

The control system is completely described by locations of sensors and control surfaces and by the transfer functions of the sensors, control laws, and actuators. Gain scheduling can be adopted by assigning different control laws to different flight conditions.

## Optimization Considerations

### Design Variables

Shape design variables have already received considerable attention in wing optimization studies.[67-71,79,80,85] However, in addition to a balanced approach to analysis in terms of the analysis techniques selected, we seek to keep a balance in level of optimization by focusing at present on sizing-type design variables in all disciplines considered (Fig. 1).

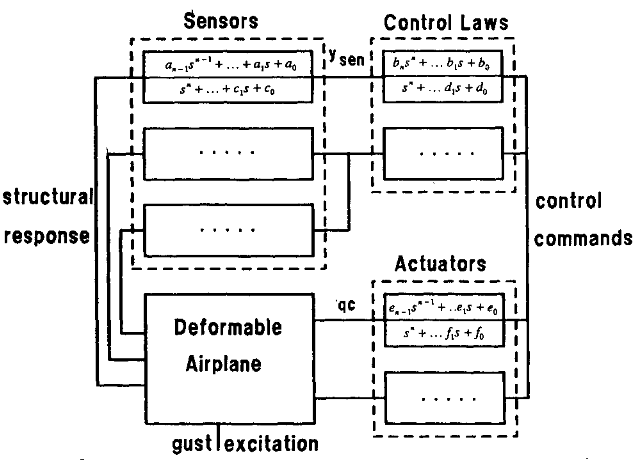


Fig. 3 Elastic airplane control system block diagram.

### Structural Design Variables

Figure 4 shows an airplane modeled as an assembly of flexible lifting surfaces. Each lifting surface is modeled as an equivalent plate whose stiffness is controlled by contributions from thin cover skins (fiber composite lamintes) and the internal structure (spar and rib caps). Wing sections are connected to each other via stiff springs (representing hinge stiffness at attach points) and flexible springs (representing the stiffness of actuators and their backup structure). Each wing section can include several trapezoidal parts. Concentrated masses are used to model nonstructural items and balance masses.

The vertical displacement $w$ of each wing section is approximated by a Ritz polynomial series of the form

$$w(x,y,t)=\sum_{i=1}^{N_w} q_i(t)x^{m_i}y^{n_i} \tag{2}$$

where $x$ and $y$ are chordwise and spanwise coordinates respectively. The exponents $m_i$ and $n_i$ define the specific polynomial series used. It can be a complete polynomial in $x$ and $y$ or a product of polynomials in $x$ and $y$ (see Ref. 98).

The depth of a wing section is given by a polynomial

$$h(x,y)=\sum_{i=1}^{N_h} H_i x^{r_i}y^{s_i} \tag{3}$$

where the $H_i$ are preassigned parameters.

Thickness distribution of a typical skin layer is represented by

$$t(x,y)=\sum_{i=1}^{N_t} T_i x^{k_i}y^{l_i} \tag{4}$$

Rib and spar cap areas are allowed to vary linearly along their length $\eta$

$$A(\eta)=A_0+A_1\eta \tag{5}$$

Wing stiffness and mass matrix elements are linear combinations of certain area integrals over the planform, line integrals over spar/rib length, and polynomial terms evaluated at points where concentrated masses are located or springs are attached.[98] The present equivalent plate modeling capability[98] makes it possible to efficiently analyze combined wing box/control surface configurations. A wing assembly and a canard or horizontal tail may be attached to a fuselage (modeled as a flexible beam or a flexible plate) to simulate complete airplane configurations. The level of modeling detail can be selected independently for each section. Therefore, the degree of detail used to model control surfaces for analysis and synthesis is not limited, as is the case of the TSO code.

At the present stage of research, structural topology, shape, and material properties are preassigned; however, skin-layer fiber orientations are available as design variables. For skin-

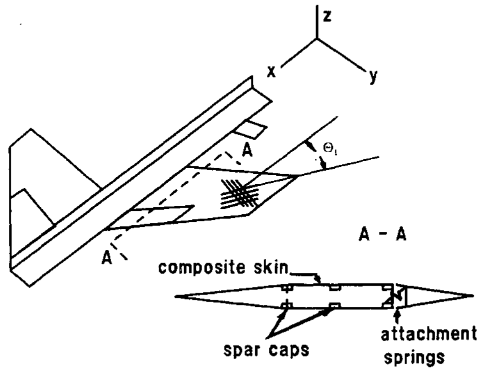


Fig. 4 Airplane as an assembly of equivalent plates.

layer thicknesses [Eq. (4)] the coefficients of the thickness power series serve as design variables. This guarantees smooth thickness variation for each layer. For spar and rib cap areas [Eq. (5)], two coefficients are used as design variables for each spar or rib. Concentrated masses at preassigned locations and spring constants for linear and rotational springs can also be treated as design variables.

## Aerodynamic and Control System Design Variables

Wing cross section, aerodynamic planform, and topology are preassigned here. Performance and loads in quasistatic maneuvers can be influenced by designing the jig shape (initial camber) of the wing and by proper deflection of leading-edge and trailing-edge control surfaces. The initial camber of the wing is given by a series

$$w^0(x,y,t) = \sum_{i=1}^{N_w} q_i^0(t)\, x^{m_i} y^{n_i} \tag{6}$$

where the powers $m_i$ and $n_i$ are identical to those in Eq. (2), and any subset of the coefficients $q_i^0$ can serve as design variables. The deflections of control surfaces for each distinct maneuver point are also available as separate design variables.

The control system design variables at the lowest level in the hierarchy (analogous to sizing) are the coefficients of numerator and denominator of control law transfer functions. Control surface locations, sensor locations, topology of the control system, and order of numerator and denominator polynomials in the transfer functions are preassigned. It is also assumed that sensor and actuator transfer functions are preassigned, although the formulation is sufficiently general so as to allow the treatment of their numerator and denominator coefficients as additional design variables.

The set of design variables treated spans three disciplines, namely structures, aerodynamics, and control. The design space is thus opened up to include sizing level design variables from all three disciplines simultaneously.

### Behavior Functions

In order to provide for a rich variety of constraints and alternative objective functions, the following analysis capabilities are included.

1) Static "maneuver load" analysis (static aeroelastic deflection and stress calculations for the elastic airplane in maneuver): maneuvers include symmetric pull-ups (defined by Mach number, altitude, and load factor) or steady rolling maneuvers (defined by Mach number, altitude, and roll rate). In addition to elastic deflections and stresses, the control surface deflections and hinge moments needed for the maneuvers are calculated.

2) Static "given loads" analysis (static deflection and stress calculations for the cantilevered wing under a set of prescribed loads): the loads are assumed independent of the structural design and do not change in the course of wing synthesis. This option is important for cases where linear aerodynamic theory is inadequate, forcing the use of experimental data.

3) Natural frequency and mode shape analysis: natural frequencies and mode shapes are obtained for different sets of boundary conditions (this facilitates generation of separate symmetric or antisymmetric modes).

4) Aeroservoelastic stability analysis: poles of the control-augmented airplane are calculated for different level flight conditions (defined by specifying Mach number and altitude).

5) Gust response analysis: root-mean-square (rms) values of control surface rotations and rates and rms values of selected sensor measurements due to continuous atmospheric turbulence are calculated for different flight conditions.

6) Drag analysis: induced drag is calculated for the elastic lift distribution during maneuvers. Drag values assuming either full leading-edge suction (fully attached flow) or no leading-edge suction (separated flow at the leading edge) or a combination of these[29,33] are available.

With the integrated analysis capability that has been developed, the following behavior functions can be evaluated: elastic displacements; elastic twist; spar/rib cap stresses; skin combined stress failure criteria[114]; natural frequencies; real and imaginary parts of aeroservoelastic poles; rms values of random control surface rotations and rates due to gust; rms values of sensor measurements in gust; total mass; lift and drag coefficients; control surface rotations; and hinge moments in maneuvers. It should also be noted that roll rate or load factor at a particular altitude and Mach number can be treated as design variables; therefore, they can be maximized or constrained.

Control surface effectiveness is not addressed directly at this stage. The synthesis emphasizes sustaining a desired roll rate or load factor while keeping hinge moments, control surface deflections, and stresses within allowable bounds.

Aeroservoelastic stability is guaranteed by providing adequate damping at each flutter critical aeroservoelastic pole throughout the flight envelope.[115] Handling qualities can be preliminarily addressed via inequality constraints on the aeroservoelastic pole locations (e.g., short-period root placement) and pilot-seat acceleration due to atmospheric turbulence.[86,89] The control surface deflection needed for trim and overall performance in a given maneuver and its rms activity due to gusts can be combined to ensure that no saturation occurs.[91,92]

Any of the behavior functions or their combinations can serve as objective functions. Possible alternatives are mass, drag (to be minimized), steady roll rate, lift-to-drag ratio (to be maximized), or a combination of these.

The present analysis capability offers a rich variety of behavior functions for wing design synthesis. Thus, the interaction among structure, control, aerodynamics, handling qualities, and airplane performance can be taken into account in an integrated manner.

### Approach to Integrated Optimization

Once the preassigned parameters, design variables, failure modes, load conditions, and objective function are selected, the integrated optimization problem can be cast as a nonlinear programming problem having the form of Eq. (1).

The nonlinear programming approach combined with approximation concepts (NLP/AC approach) has proven to be an effective method for solving structural synthesis problems,[21,22] and here it will be adapted to the multidisciplinary design optimization task. In this method relatively few detailed analyses are carried out during optimization. Each analysis and the associated behavior sensitivity analysis serve as a basis for constructing approximations to the objective and constraint functions in terms of the design variables. Thus, a series of explicit approximate optimization problems is solved converging to an optimal design.

The main advantage of the NLP formulation is its generality. No a priori assumptions have to be made about the set of active constraints at the optimum. Given an initial design, a local optimum is sought using mathematical programming techniques. Thus, it is especially suitable for multidisciplinary optimization, where the problem is large and complicated and past experience does not provide much intuitive guidance. However, for the NLP approach to be practical, it is crucial to avoid too many detailed analyses. Success in this regard depends on efficient analysis/sensitivity calculations and on making the explicit approximations of objective and constraint functions robust yet simple enough for efficient solution.

The use of analytic behavior sensitivities and the construction of robust approximations for behavior functions in terms of the design variables are at the heart of the NLP/AC approach. During the past two decades, approximation techniques for static deflection, stress, and natural frequency constraints have been studied extensively and are now well established. Several methods for divergence and flutter constraint treatment have been developed, but no experience has been reported with the aeroservoelastic poles of an actively controlled airplane, the rms of the response to random gusts, or hinge moments of a flexible control surface on an elastic wing. It should be pointed out that in maneuver load analysis the loads acting on the airplane are functions of the design variables via static aeroelasticity as well as through inertial effects. The right side $\{P\}$ of the matrix equation

$$[\bar{K}]\{q\} = \{P\} \tag{7}$$

(where $\bar{K}$ is a stiffness matrix modified by aerodynamic terms corresponding to structural and control surface motions) thus depends on the structural design variables, whereas in the classical given loads analysis, the right side $\{P\}$ is fixed. Thus, the success of approximations using reciprocal variables (Ref. 116) for stress and static deflection in static analysis is not guaranteed in maneuver load analysis, and alternative approximations have to be carefully examined.

Some aspects of maneuver load and drag analysis have already been discussed in the literature within the framework of integrated wing optimization.[29,33,67,68,70] In the following we will, therefore, focus on the aeroservoelastic and gust response analysis and behavior sensitivity calculations.

## Aeroservoelastic Analysis

As pointed out in Ref. 117, all time-dependent phenomena of the elastic airplane are governed by a universal set of equations of motion, wherein only the right side (representing the input) varies in proceeding from one phenomenon to the next. Indeed a measure of multidisciplinary integration of analysis techniques, concepts, and terminology is needed even before multidisciplinary optimization is addressed.

Several steps in this direction have been taken in the past.[118-121] However, almost 20 years after the publication of Ref. 118, still no particular approach to the dynamics of the deformable airplane is universally accepted. The analysis here for time-dependent problems is based on the widely used set of linearized equations for small perturbations of elastic airplane motion with respect to constant speed, level flight.[122-127] Although perturbations in the longitudinal direction are not taken into account at this stage, the analysis is adequate for addressing basic stability and control as well as aeroservoelastic problems of highly augmented airplanes in the context of preliminary design.[89]

### Aeroelastic Model

Assuming small perturbations from a steady level flight, the Laplace transformed equations of motion for the elastic airplane are

$$\{[M]s^2 + [C]s + [K]\}\{q(s)\} - q_D S[Q(s)]\{q(s)\} = q_D S\{Q_G(s)\}\frac{w_G(s)}{U_\infty} \tag{8}$$

The vector of displacements can be expressed in terms of $n_s$ structural response DOF and $n_c$ control surface deflections:

$$\{q\} = \begin{Bmatrix} q_s \\ q_c \end{Bmatrix} \tag{9}$$

and the equations of motion corresponding to the structural DOF are partitioned accordingly to yield

$$\{[M_{ss}]s^2 + [C_{ss}]s + [K_{ss}]\}\{q_s\} + \{[M_{sc}]s^2 + [C_{sc}]s + [K_{sc}]\}\{q_c\} - q_D S[Q_{ss}(s)]\}\{q_s\} - q_D S[Q_{sc}(s)]\}\{q_c\} = \frac{q_D}{U_\infty} S\{Q_{Gs}(s)\}w_G(s) \tag{10}$$

The finite-dimensional approximation to the Laplace transformed generalized unsteady loads using the Roger approach[111,113] is of the form

$$[Q(s)] = s^2[P_1] + s[P_2] + [P_3] + \sum_{l=1}^{l=N_l}\frac{s}{(s+b_l)}[P_{l+3}] \tag{11}$$

In the minimum state method,[112,113] the functional dependence of the generalized aerodynamic force matrix on the Laplace variable is approximated by a rational expression of the form

$$[Q(s)] = [P_1]s^2 + [P_2]s + [P_3] + [P_4]\Big[s[I] - [P_5]\Big]^{-1}[P_6]s \tag{12}$$

The finite-dimensional Roger approximation of the Laplace transformed generalized unsteady gust loads used is of the form

$$[Q_G(s)] = s[P_2^G] + [P_3^G] + \sum_{l=1}^{l=N_l}\frac{s}{(s+b_l^G)}[P_{l+3}^G] \tag{13}$$

The minimum state approximation used for the gust load vector is

$$[Q_G(s)] = [P_2^G]s + [P_3^G] + [P_4^G]\Big[s[I] - [P_5^G]\Big]^{-1}[P_6^G]s \tag{14}$$

(Note that the notation $[P]$ is used to denote matrices associated with unsteady aerodynamics finite-dimensional state space approximations. However, the matrices $[P_l]$ and $[P_l^G]$ ($l \geq 4$) have different meanings depending on whether the Roger or minimum state approximation is used.)

Partitioning the aerodynamic matrices associated with structural DOF according to Eq. (9) leads to a Roger approximation of the form

$$[Q^{ss}(s), Q^{sc}(s)] = s^2[P_1^{ss}, P_1^{sc}] + s[P_2^{ss}, P_2^{sc}] + [P_3^{ss}, P_3^{sc}] + \sum_{l=1}^{l=N_l}\frac{s}{(s+b_l)}[P_{l+3}^{ss}, P_{l+3}^{sc}] \tag{15}$$

and to a minimum state partitioned approximation of the form

$$[Q^{ss}(s)] = [P_1^{ss}]s^2 + [P_2^{ss}]s + [P_3^{ss}] + [P_4^s][sI - P_5]^{-1}[P_6^s]s \tag{16}$$

$$[Q^{sc}(s)] = [P_1^{sc}]s^2 + [P_2^{sc}]s + [P_3^{sc}] + [P_4^s][sI - P_5]^{-1}[P_6^c]s \tag{17}$$

In the above expressions, $[P_1^{ss}], [P_2^{ss}], [P_3^{ss}]$ are aerodynamic apparent mass, damping, and stiffness matrices, respectively. Their dimensions are $(n_s \times n_s)$. $[P_1^{sc}], [P_2^{sc}], [P_3^{sc}]$ are aerodynamic apparent mass, damping, and stiffness matrices, respectively, associated with the coupling terms between control surface and structural motion. Their dimensions are $(n_s \times n_c)$.

Aerodynamic states are now introduced as follows. For the Roger approximation

$$\{r_l^A(s)\} = \frac{s}{(s+b_l)}[P_{l+3}^{ss}, P_{l+3}^{sc}]\begin{Bmatrix} q_s \\ q_c \end{Bmatrix} \tag{18}$$

$$\{r_l^G(s)\} = \frac{s}{(s+b_l^G)}\{P_{l+3}^G\}w_G(s) \tag{19}$$

Aerodynamic states for the minimum state method are introduced as

$$\{r_s^A(s)\} = \Big[s[I] - [P_5]\Big]^{-1}[P_6^s\ P_6^c]s\begin{Bmatrix} q_s(s) \\ q_c(s) \end{Bmatrix} \tag{20}$$

$$\{r_s^G(s)\} = \Big[s[I] - [P_5^G]\Big]^{-1}[P_6^G]w_G(s) \tag{21}$$

Thus, in the Roger approximation, the state space equations for the added aerodynamic states associated with structural deflection are

$$s\{r_l^A(s)\} = -b_l\{r_l^A(s)\} + [P_{l+3}^{ss}]s\{q_s\} + [P_{l+3}^{sc}]s\{q_c\} \tag{22}$$

and in the minimum state approximation

$$s\{r_s^A(s)\} = [P_5]\{r_s^A(s)\} + [P_6^s]s\{q_s(s)\} + [P_6^c]s\{q_c(s)\} \tag{23}$$

Two vertical wind gust state space models are available in the literature for random gust response calculations. These are the Dryden[5,6] model and any rational approximation to the von-Karman[128,129] model. In both cases a gust filter, represented by a strictly proper transfer function, is used to transform a Gaussian zero mean white noise input $w$ into the vertical gust speed $w_G$ with a given power spectral density and rms. A state space description of the gust input is thus of the form

$$s\{x_G\} = [A_G]\{x_G\} + \{B_G\}w \tag{24}$$

$$w_G(s) = [C_G]\{x_G\} \tag{25}$$

$$s\ w_G(s) = [C_G][A_G]\{x_G\} + [C_G]\{B_G\}w \tag{26}$$

If aerodynamically augmented mass, damping, and stiffness matrices are now introduced in the form

$$\hat{M}_{ss} = M_{ss} - q_D S P_1^{ss} \tag{27}$$

$$\hat{C}_{ss} = C_{ss} - q_D S P_2^{ss} \tag{28}$$

$$\hat{K}_{ss} = K_{ss} - q_D S P_3^{ss} \tag{29}$$

$$\hat{M}_{sc} = M_{sc} - q_D S P_1^{sc} \tag{30}$$

$$\hat{C}_{sc} = C_{sc} - q_D S P_2^{sc} \tag{31}$$

$$\hat{K}_{sc} = K_{sc} - q_D S P_3^{sc} \tag{32}$$

($K_{sc}$ and $C_{sc}$ are zero)

and five subvectors of the state vector $\{x\}$ are defined as

$$\{x_1\} = \{q_s\} \tag{33}$$

$$\{x_2\} = s\{q_s\} \tag{34}$$

$$\{x_3\} = \{x_G\} \tag{35}$$

$$\{x_4\} = \{r^A\} \tag{36}$$

$$\{x_5\} = \{r^G\} \tag{37}$$

then after substitution of Eqs. (11-32) into Eq. (10), the structural and unsteady aerodynamic part of the integrated equations of motion can be written in matrix form as

$$s[E]\{x\} = [F]\{x\} + [G]\{u\} + \{H\}w \tag{38}$$

where $\{u\}$ is a vector of control surface excitations

$$\{u\} = \begin{Bmatrix} q_c \\ sq_c \\ s^2q_c \end{Bmatrix} \tag{39}$$

while $[E]$,$[F]$,$[G]$, and $\{H\}$ are matrices whose elements depend on structural, aerodynamic, and gust filter terms. Equations (38) are the Laplace transformed equations for the structural and aerodynamic states. To complete the state space formulation of this problem, the feedback expression relating the control surface motion $\{q_c(s)\}$ to the structural motions $\{q_s(s)\}$, $\{sq_s(s)\}$, $\{s^2q_s(s)\}$ is used to close the control loops.

**Control System**

The actual displacements, velocities, and accelerations at a set of points on the structure are given by

$$\{y_{STR}(s)\} = \{[\Phi_0] + [\Phi_1]s + [\Phi_2]s^2\}\{q_s(s)\} \tag{40}$$

The matrices $\Phi_0$, $\Phi_1$, $\Phi_2$ are determined by the location of measurement points on the structure. Actual structural responses are measured by a set of sensors whose output signals serve as input to control laws. These control laws generate input commands $\{\delta\}$ to the actuators. The $n_c$ commands, $\delta_i$, are synthesized to serve as input to $n_c$ actuators.

A typical control law transfer function is given by

$$\frac{\delta_i}{y_{SEi}} = \frac{b_n s^n + b_{n-1}s^{n-1} + \ldots.. + b_0}{s^n + d_{n-1}s^{n-1} + \ldots.. + d_0} \tag{41}$$

where $y_{SEi}$ is the input signal to the control element. The state equations for a single control law are given by

$$s\{x_{CO}\} = \begin{bmatrix} 0 & 0 & . & -d_0 \\ 1 & 0 & . & -d_1 \\ 0 & 1 & . & -d_2 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & . & -d_{n-1} \end{bmatrix}\{x_{CO}\} + \left\{ b_n \begin{bmatrix} d_0 \\ d_1 \\ . \\ . \\ . \\ . \\ d_{n-1} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ . \\ . \\ . \\ . \\ b_{n-1} \end{bmatrix} \right\} y_{SEi} \tag{42}$$

$$\delta_i = \{0\,0\,0 \cdot\cdot\cdot -1\}\{x_{CO}\} + b_n\, y_{SEi} \tag{43}$$

where $d_i$ and $b_i$ are the transfer function denominator and numerator coefficients. When all control laws are assembled, the multi-input multi-output control block controlling several actuators is represented by

$$s\{x_{CO}\} = [A_{CO}]\{x_{CO}\} + [B_{CO}]\{y_{SE}\} \tag{44}$$

$$\{\delta\} = [C_{CO}]\{x_{CO}\} + [D_{CO}]\{y_{SE}\} \tag{45}$$

Sensor and actuator transfer functions are assumed to have denominators of higher order than their numerators. When several sensors are present, the state space model relating the sensor states $\{x_{SE}\}$, the sensor outputs $\{y_{SE}\}$, and the actual structural response $\{y_{STR}\}$ is

$$s\ \{x_{SE}\} = [A_{SE}]\{x_{SE}\} + [B_{SE}]\{y_{STR}\} \tag{46}$$

$$\{y_{SE}\} = [C_{SE}]\{x_{SE}\} \tag{47}$$

The state space model relating the sensor states $\{x_{SE}\}$, the sensor outputs $\{y_{SE}\}$, and the actual response $\{y_{STR}\}$ is [see Eqs. (33), (34), (40), and (47)].

$$s\,\{x_{SE}\} = [A_{SE}]\{x_{SE}\} + [B_{SE}]([\Phi_0]\{x_1\} + [\Phi_1]\{x_2\} + [\Phi_2 s]\{x_2\}) \tag{48}$$

The state space model of the actuators is given by

$$s\,\{x_{ACT}\} = [A_{ACT}]\{x_{ACT}\} + [B_{ACT}]\{\delta\} \tag{49}$$

$$\{q_c\} = [C_{ACT}]\{x_{ACT}\} \tag{50}$$

where $\{q_c\}$ is the control surface deflection (assuming irreversible surfaces). The matrices $[A_{CO}]$, $[B_{CO}]$, $[D_{CO}]$, $[A_{SE}]$, $[B_{SE}]$, $[A_{ACT}]$, and $[B_{ACT}]$ depend on the design variables associated with the control system through relations similar to Eqs. (42) and (43). Using Eqs. (44–50), the control surface excitation vector $\{u\}$ [see Eq. (39)] can now be expressed in terms of structural, aerodynamic, and control system states as follows:

$$\{q_c\} = [C_{ACT}]\{x_{ACT}\} \tag{51}$$

$$s\{q_c\} = [C_{ACT}][A_{ACT}]\{x_{ACT}\} + [C_{ACT}][B_{ACT}][C_{CO}]\{x_{CO}\} + [C_{ACT}][B_{ACT}][D_{CO}][C_{SE}]\{x_{SE}\} \tag{52}$$

$$\begin{aligned} s^2\,\{q_c\} &= [C_{ACT}][B_{ACT}][D_{CO}][C_{SE}][B_{SE}] \\ &\times ([\Phi_0]\{q_s\} + [\Phi_1]s\{q_s\} + [\Phi_2]s^2\{q_s\}) \\ &+ [C_{ACT}][A_{ACT}]^2\{x_{ACT}\} + [C_{ACT}]([A_{ACT}][B_{ACT}][D_{CO}][C_{SE}] \\ &+ [B_{ACT}][C_{CO}][B_{CO}][C_{SE}] + [B_{ACT}][D_{CO}][C_{SE}][A_{SE}]) \\ &\times \{x_{SE}\} + [C_{ACT}]([A_{ACT}][B_{ACT}][C_{CO}] \\ &+ [B_{ACT}][C_{CO}][A_{CO}])\{x_{CO}\} \end{aligned} \tag{53}$$

**Complete System**

The state vector of the whole system is partitioned into eight subvectors. The following subvectors are added to the five which have been already defined in Eqs. (33–37):

$$\{x_6\} = \{x_{ACT}\}(n_{ACT} \times 1) \tag{54}$$

$$\{x_7\} = \{x_{SE}\}(n_{SE} \times 1) \tag{55}$$

$$\{x_8\} = \{x_{CO}\}(n_{CO} \times 1) \tag{56}$$

Assembly of the state space models of the structure, sensors, actuators, control block, and approximate unsteady load and gust aerodynamics leads to the closed-loop state space equations of the complete system:

$$s\,[U]\{x(s)\} = [V]\{x(s)\} + \{W\}w \tag{57}$$

The order of the system is given by

$$n_{SYS} = 2\,n_s + n_{ACT} + n_{SE} + n_{CO} + n_G + n_{AER} \tag{58}$$

When $N_l$ lag terms are used, then in the case of Roger approximation

$$n_{AER} = N_l\, n_s + N_l^G$$

and in the case of minimum state approximation

$$n_{AER} = N_l + N_l^G$$

The considerable saving in terms of added aerodynamic states with the minimum state approach is now evident, since with the Roger approximation the number of added states is a product of the number of lag terms used to fit the data [Eq. (11)] times the number of structural DOF used.

**Dependence of System Matrices on the Design Variables**

Examine the $[E]$, $[F]$, $[G]$, and $[H]$ matrices of Eq. (38). When augmented by the control system state space equations for the control system states [Eqs. (44), (48), and (49)] and if full-order structural matrices are used (no modal reduction), then each of these matrices is a linear combination of structural, aerodynamic, and products of control system terms. The transformation matrix relating system states $\{x\}$ to control excitations $\{u\}$ [Eqs. (51–53)] depends only on control system parameters (no modal reduction). Substitution of Eqs. (51–53) into Eq. (38) shows that the elements of the $[U]$, $[V]$, and $\{W\}$ matrices of Eq. (57) are of the form

$$(S1)_{ij} + (A1)_{ij} + (C1)_{ij} + (S2)_{ij}(C2)_{ij} + (A2)_{ij}(C3)_{ij} \tag{59}$$

The structural mass and stiffness terms $S1$, $S2$ depend only on structural design variables, whereas the control system matrices $C1$, $C2$, $C3$ depend only on control system design variables via the state space models of actuators, sensors, and control laws. Since wing cross section and planform shape are preassigned at present, when full-order stiffness and mass matrices are used, the aerodynamic terms associated with generalized unsteady loads and gust forces do not change with a change in design variables. When modal reduction is used to reduce the order of the system matrices, then a fixed modes approach is adopted here[130] resulting in fixed aerodynamic terms for the modally reduced models also. Modal reduction is further discussed in the next section.

**Full-Order and Reduced-Order Models**

With the equivalent plate approach, the structural model may include a relatively small number of DOF when compared to conventional finite element models. These are generalized displacements associated with the Ritz polynomials. The generalized aerodynamic matrices are calculated for the same set of Ritz polynomials used in Eq. (2). Thus, the aeroservoelastic stability analysis can be done with a full-order model, including the full-order mass, stiffness, and aerodynamic matrices or by order reduction based on a small number of normal modes.

If a Roger or a minimum state approximation can be found that will accurately fit the full-order aerodynamic matrices with a small number of lag terms, then these approximation matrices can be used in the aeroservoelastic stability analysis even with modal reduction. They just have to be premultiplied and postmultiplied by the generalized mode shapes in order to have the approximation to the modally reduced aerodynamic matrices. Although the number of structural DOF might differ significantly between full-order and modal analysis, the number of aerodynamic states is the same when both approaches are based on the same minimum state approximation to the full-order aerodynamic matrices. If the Roger approximation is used, the number of added aerodynamic states increases with any increase in the number of modes used for reduction. This produces very large models when many modes or a full-order analysis are used. Another possibility is first to modally reduce the frequency-dependent unsteady aerodynamic matrices using modes that are periodically updated after a given number of analysis/sensitivity optimization cycles. Then it might be possible to fit these reduced matrices using fewer lag terms than the number needed to fit the full-order matrices. If a smaller number of lag terms can thus be used, this will reduce the dimensions of the $U$, $V$ matrices in Eq. (57), compared to their dimensionality in a modal approach, which uses pregenerated full-order aerodynamic matrices. However, rather than preparing the full-order aerodynamic approximantes only once before any synthesis starts, in the latter case it would be necessary to generate aerodynamic approximants each time the set of modes used as a basis for modal reduction is changed during the synthesis. All of the modal reduction/unsteady aerodynamic approximation alternatives described so far have been included in the present capability. However, their comparative assessment is beyond the scope of this paper.

When full-order aerodynamic matrices are used, they are generated once for the Ritz functions employed in the structural analysis, and they are invariant with respect to changes in either structural sizing or control system design variables. When modal reduction is used, the mode shapes are periodically updated after a given number of analysis steps and so are the aerodynamic matrices and their finite-dimensional rational approximations. Nevertheless, in eigenvalue sensitivity calculations, the derivatives of all reduced-order matrices are determined using a fixed-mode approach.[130] This might require the use of more modes in order to obtain good sensitivity information. In summary, the derivatives of the aerodynamic matrices with respect to structural or control system design variables are zero for the full-order case, and they are assumed zero for the reduced-order case.

The derivatives $\partial U/\partial p$, $\partial V/\partial p$ can be calculated analytically for either structural or control system design variables ($p$) by differentiting Eq. (59). The gust load vector $\{W\}$ depends only on gust filter and aerodynamic design variables, and therefore its partial derivative with respect to $p$ vanishes.

Stability is examined by computing the eigenvalues of the generalized eigenvalue problem:

$$\lambda[U(p)]\{\phi\} = [V(p)]\{\phi\} \tag{60}$$

Sensitivity of eigenvalues with respect to structural and control system design variables is given by

$$\frac{\partial\lambda}{\partial p} = \frac{\psi^T\left[\lambda\dfrac{\partial U}{\partial p} - \dfrac{\partial V}{\partial p}\right]\phi}{\psi^T V\phi} \tag{61}$$

where $\psi$ and $\phi$ are left and right eigenvectors, respectively.

#### Gust Response Analysis and Sensitivity

In addition to several publications addressing it in the context of active control technology,[5,6,39] airplane response to random atmospheric turbulence has already been discussed in the context of structural optimization.[131,132] Here, attention is focused on the rms values of control surface rotations $\{q_c\}$, rates $\{\dot{q}_c\}$, and sensor measurements $\{y_{SE}\}$.

The state space equations [Eq. (57)] are transformed into standard form

$$s\,\{x(s)\} = [\tilde{A}]\{x(s)\} + \{\tilde{B}\}w(s) \tag{62}$$

Since only $\{q_c\}$, $s\{q_c\}$, and $\{y_{SE}\}$ are considered, [Eqs. (47), (51), and (52)], it follows that each output considered $y_k$ is given by

$$y_k = \{C_k\}^T\{x\} \tag{63}$$

where $\{C_k\}$ is either constant or a function of control system design variables.

The state covariance matrix is a solution of a Lyapunov's matrix equation[133] in the form

$$[\tilde{A}][X] + [X][\tilde{A}]^T = -\{\tilde{B}\}[Q_w]\{\tilde{B}\}^T \tag{64}$$

where $[Q_w]$ is the intensity matrix of the Gaussian white noise $w$. Sensitivity of the covariance matrix $[X]$ with respect to a design variable $p$ is calculated by differentiating Eq. (64):

$$[\tilde{A}]\left[\frac{\partial X}{\partial p}\right] + \left[\frac{\partial X}{\partial p}\right][\tilde{A}]^T = -2\{\tilde{B}\}[Q_w]\left\{\frac{\partial \tilde{B}}{\partial p}\right\}^T - [X]\left[\frac{\partial \tilde{A}}{\partial p}\right]^T - \left[\frac{\partial \tilde{A}}{\partial p}\right][X] \tag{65}$$

and finally the derivative of the (rms)$^2$ of $y_k$ is calculated by differentiation of the covariance expressions based on Eqs. (47), (51), and (52). It should be noted that

$$[\tilde{A}] = [U]^{-1}[V] \tag{66}$$

$$\{\tilde{B}\} = [U]^{-1}\{W\} \tag{67}$$

and

$$\frac{\partial[U]^{-1}}{\partial p} = [U]^{-1}\left[\frac{\partial U}{\partial p}\right][U]^{-1} \tag{68}$$

from which it follows that

$$\left[\frac{\partial \tilde{A}}{\partial p}\right] = [U]^{-1}\left[\frac{\partial V}{\partial p} - \frac{\partial U}{\partial p}\tilde{A}\right] \tag{69}$$

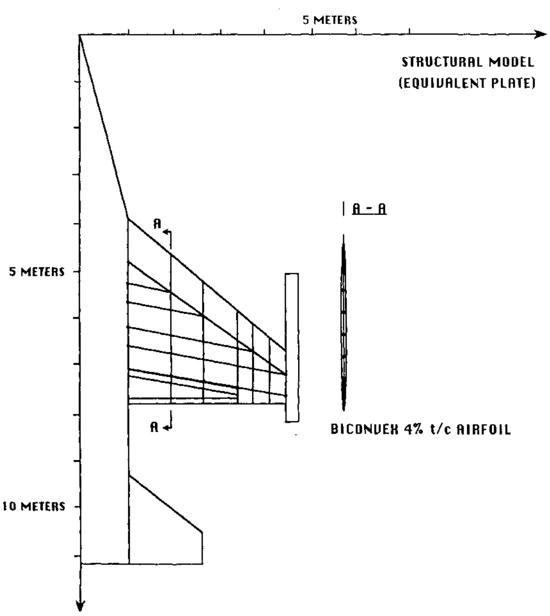


**Fig. 5 Lightweight fighter structural model.**

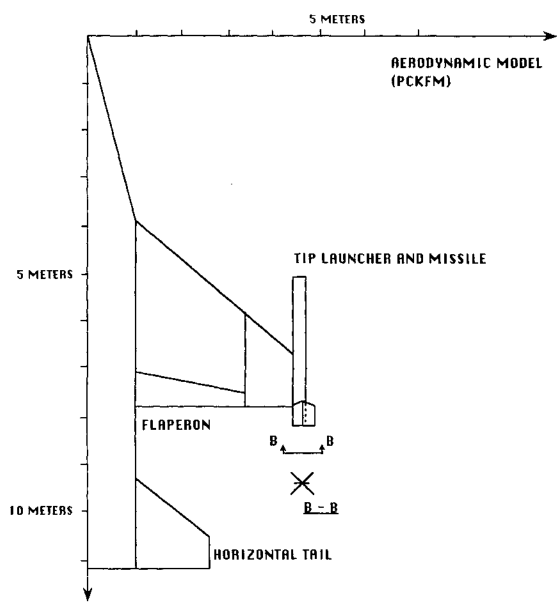


**Fig. 6 Lightweight fighter aerodynamic model.**

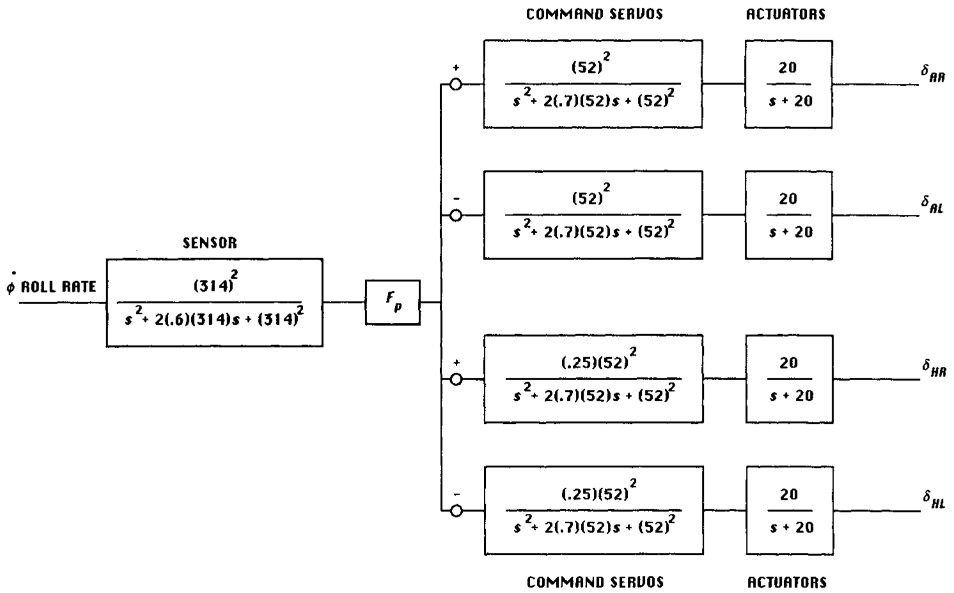


Fig. 7 Lightweight fighter control system roll channel.

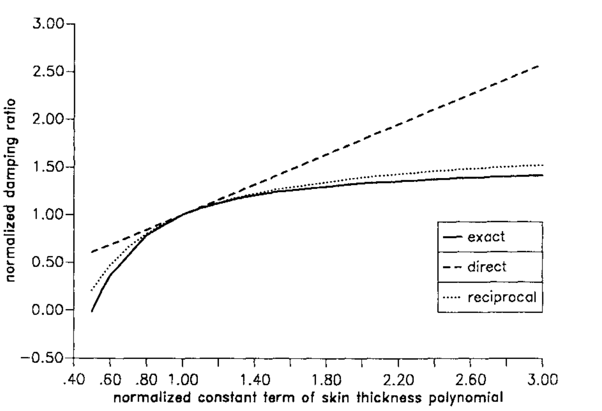


**Fig. 8 Approximations of missile pitch root damping (variable wing thickness).**

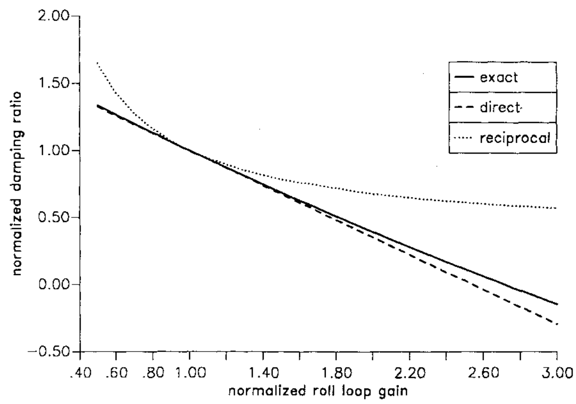


**Fig. 9 Approximations of missile pitch root damping (variable roll loop gain).**

and

$$\left\{\frac{\partial \tilde{B}}{\partial p}\right\} = [U]^{-1}\left[\frac{\partial W}{\partial p} - \frac{\partial U}{\partial p}\tilde{B}\right] \tag{70}$$

The Hessenberg-Schur method of Ref. 134 is used to solve both Eqs. (64) and (65). Note that after the analysis is carried out, the matrices $[\tilde{A}]$, $[\tilde{A}]^T$ are already reduced to Hessenberg and Schur forms. Therefore, the sensitivity calculations of Eq. (65) are equivalent to adding right sides to Eq. (64).

## Numerical Examples

### Actively Controlled Lightweight Fighter Test Model

Structural and aerodynamic models of a lightweight fighter airplane are shown in Figs. 5 and 6. The airplane is similar to the YF-16, and the construction of its mathematical model was guided by Refs. 29, 135, and 136. It is different than an actual YF-16. Vertical tail and ventral fins are not included. The fuselage and horizontal tail are assumed rigid. There is no leading-edge flap. The wing has a biconvex (about 4% thickness-to-chord ratio) airfoil. It is made of aluminum skins and an array of spars and ribs. A flexible aileron and a rigid tip launcher/missile assembly are attached to the wing using springs. The configuration analyzed weighes 20,000 lb and is statically unstable.

The roll channel of the flight control system is shown in Fig. 7. It is based on the YF-16 roll channel (Refs. 12, 135, and 136). The lightweight fighter model used here is intended to illustrate the well-known YF-16 aeroservoelastic roll instability (Refs. 12, 13, and 135). Lack of sufficient data precluded a more refined simulation of the YF-16. However, the model used here is representative of a typical fighter airplane and is quite complex in its detail.

Two mechanisms of instability, similar to those encountered in the YF-16, exist here. As the aileron gain $F_p$ (Fig. 7) is increased, a 6.5 Hz instability appears associated with a missile pitch mode. With a further increase of $F_p$, a second instability appears at 3.5 Hz associated with the rigid-body roll mode. Mach 0.9 aerodynamics for antisymmetric motion at 20,000 ft is used in the stability calculations. Since the original airplane is unstable at this point, it is artifitially stabilized here by reducing the gain $F_p$ and adding inherent damping. Stability of the model is necessary for studying gust response approximations.

## Results

The multidisciplinary wing analysis/synthesis capability described here has been extensively tested. Structural, aerodynamic, aeroelastic, and aeroservoelastic results have been compared with analysis/test data available from other sources, and overall good correlation was found. The analytic behavior sensitivities were verified by finite-difference sensitivity checks. Here we emphasize some of the basic issues associated with the feasibility of applying the NLP/AC approach to truly integrated multidisciplinary wing synthesis.

These are the computational speed of analysis/sensitivity calculations and the robustness of behavior function approximations.

Parametric studies of the effects of design variable variations on aeroservoelastic poles are presented in Figs. 8 and 9. Figure 8 shows the variation of the damping ratio $\zeta$ of the missile pitch mode with a typical structural thickness design variable. In Fig. 9 the damping varies with a typical control system design variable: the gain $(F_p)$ (Fig. 7). In both figures the design variable is varied over a wide range of values. In practical optimization, move limits will be imposed on design variables to ensure accuracy of behavior function approximations. When move limits of 30% are used, both direct and reciprocal Taylor series representations[116] yield reasonable approximations. Thus, hybrid approximations seem adequate for explicit representations of aeroservoelastic pole constraints.

Figure 10 shows the variation of the aileron mean square deflection due to atmospheric turbulence when the gain $F_p$ is varied. As the missile pitch mode is destabilized (Fig. 9), the gust response increases sharply. Away from the stability boundary, the mean square (ms) aileron deflection is well-approximated by either a linear or reciprocal approximation with 30% move limits. Tighter move limits might be needed near the stability boundary as damping ratios approach zero. It is expected that any optimization procedure will respond to a decrease in damping and an increase in gust response by driving the design away from damping and gust response constraint violations.

Constraints associated with maneuver loads are evaluated next to determine the quality of approximations [see Eq. (7)]. The thickness of the skin on the wing inboard box is varied over a wide range of values. The variation of the aileron hinge moment needed to sustain a desired steady-state roll at sea level, $M = 0.9$, is shown in Fig. 11. A stress constraint for a point on the skin in a 9 $g$ symmetric pull-up in terms of a quadratic stress failure criterion[114] is illustrated in Fig. 12. The aileron hinge moment increases as aileron effectiveness is lost due to a decrease in wing skin thickness in Fig. 11. Again, with move limits of 30%, both linear and reciprocal approximations work quite well. Based on the examples presented here, it is expected that robust approximations can be constructed for constraints associated with a) deflection, stress, control surface trim angle, and hinge moment in given pull-up and roll maneuvers and b) aeroservoelastic stability and gust response limitations in given level flight conditions. Similar considerations can be expected to apply to deflection and stress constraints in "given-load" load conditions as well as natural frequency constraints based on the extensive study of these constraints within the framework of structural synthesis.[116]

Typical CPU times for execution of an analysis/sensitivity cycle on the lightweight fighter are given in Table 1. In this example, three maneuver load cases and one given-load case, symmetric and antisymmetric modes are calculated, and aeroservoelastic stability and gust response are included for one level flight condition. The wing is covered by a grid of 81 points for deflection and stress calculations. A total of 3222 constraints and all of their sensitivities with respect to 40 structural, aerodynamic, and control system design variables are calculated in about 7 min. The constraints include a comprehensive mix of gauge, slope, displacement, stress, natural frequency, aeroservoelastic pole, gust response, drag, hinge moment, and control surface travel constraints. As Table 1 shows, the major part of the computation time in one analysis/sensitivity cycle is spent on the behavior sensitivity calculations. Constraint deletion[22] strategies will reduce this time considerably by retaining only a small subset of critical and potentially critical constraints as drivers in each approximate problem formulation. Only sensitivities of the retained constraints are needed, and CPU time for one detailed analysis/sensitivity/approximate problem generation stage will be reduced considerably. Thus, if about 10–15 detailed analyses are

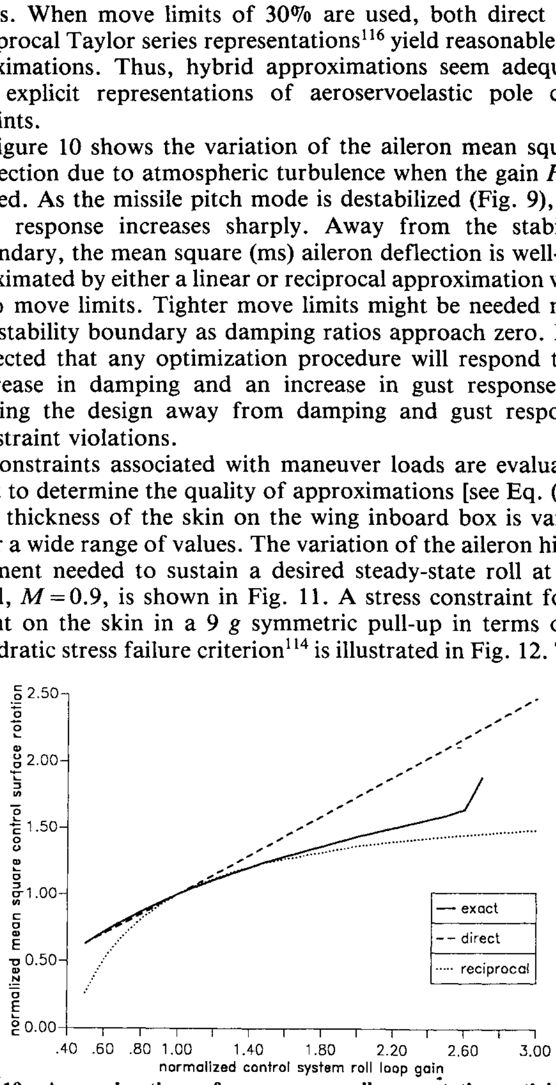


**Fig. 10 Approximations of mean square aileron rotation activity in turbulence (variable roll loop gain).**

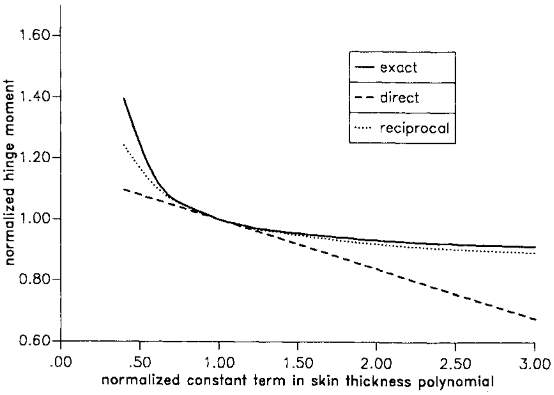


**Fig. 11 Approximations of hinge moment in steady roll (variable wing thickness).**

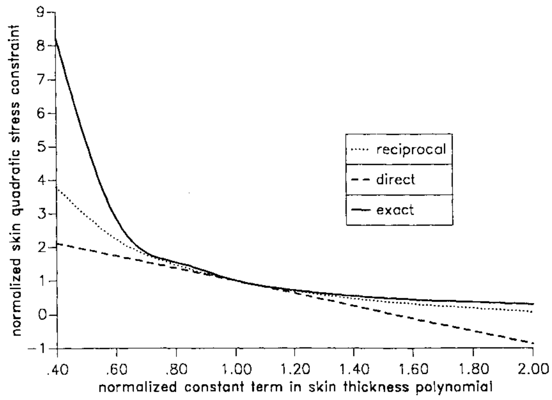


**Fig. 12 Approximations of skin quadratic stress failure criterion in a 9 g symmetric pull-up maneuver (variable wing thickness).**

**Table 1 Typical computation times, seconds**

| | |
|---|---|
| Generate $M$, $K$, and $\{P\}$ | 5.4 |
| Given-load solution | 0.09 |
| Maneuver load solutions | 0.35 |
| Drag calculations | 0.07 |
| Natural modes | 4.8 |
| Generate $A$, $B$ matrices | 1.53 |
| Aeroservoelastic stability analysis | 15.58 |
| Gust response analysis | 11.44 |
| Deflection, stresses calculations | 0.86 |
| Total analysis | 40.12 |
| Stess, deflection sensitivities | 295.15 |
| Natural frequency sensitivities | 1.47 |
| Aeroservoelastic pole sensitivities + gust response sensitivities | 53.15 |
| Total sensitivity | 349.77 |

needed for optimization based on approximation concepts (Ref. 22), it can be anticipated that between 40-60 CPU min will be needed on the UCLA IBM 3090 Model 200 for integrated multidisciplinary optimization of practical wings.

## Concluding Remarks

A general framework for wing optimization has been developed, highlighting the multidisciplinary nature of the problem. A balanced multidisciplinary wing analysis and behavior sensitivity analysis capability has been described. Emphasis was placed on various aspects of the aeroservoelastic problem formulation as well as integration and testing of the aeroelastic elements of the new method. Promising results in terms of approximation accuracy and computation times indicate that the integrated multidisciplinary optimization of practical actively controlled, fiber composite wings is within reach.

## Acknowledgment

This research was supported by AFOSR Contract F49620-87-K-0003.

## References

[1]Hwang, C., and Kesler, D. F., "Aircraft Active Controls – New Era in Design," *Astronautics & Aeronautics*, June 1983, pp. 70-85.

[2]Hanson, P. W., "An Aeroelastician's Perspective of Wind Tunnel and Flight Experiences with Active Control of Structural Response and Stability," NASA TM-85761, April 1984 (available from NTIS as N84-23924).

[3]Newsom, J. R., Adams, W. M., Mukhopadhyay, V., Tiffany, S. H., and Abel I., "Active Controls: A Look at Analytical Methods and Associated Tools," *Proceedings of the 14th Congress of the International Council of the Aeronautical Sciences*, Toulouse, France, 1984, ICAS Paper 84-42.3.

[4]Nissim, E., "Design of Control Laws for Flutter Suppression Based on the Aerodynamic Energy Concept and Comparisons with Other Design Methods," AIAA Paper 89-1212, April 1989.

[5]Liebst, B. S., Garrard, W. L., and Farm, J. A., "Design of a Multivariable Flutter Suppression/Gust Load Alleviation System," *Journal of Guidance, Control and Dynamics*, Vol. 11, No. 3, May-June 1988, pp. 220-229.

[6]Schmidt, D. K., and Chen, T. K., "Frequency Domain Synthesis of a Robust Flutter Suppression Control Law," *Journal of Guidance, Control and Dynamics*, Vol. 9, No. 3, May-June 1986, pp. 346-351.

[7]Lynch, R. W., and Rogers, Q. A., "Aeroelastic Tailoring of Composite Materials to Improve Performance," *Proceedings of the 16th AIAA Structures, Structural Dynamics and Materials Conference*, 1975, pp. 61-68.

[8]McCullers, L. A., "Automated Design of Advanced Composite Structures," *Mechanics of Composite Materials*, edited by Z. Hashin, Pergamon Press, New York, 1983.

[9]Lansing, W., Lerner, E., and Taylor, R. F., "Applications of Structural Optimization for Strength and Aeroelastic Design Requirements," AGARD-R-664, London, 1978.

[10]Shirk, M. H., Hertz, T. J., and Weisshaar, T. A., "Aeroelastic Tailoring—Theory, Practice, Promise," *Journal of Aircraft*, Vol. 23, No. 1, Jan. 1986, pp. 6-18.

[11]Weisshaar, T. A., "Aeroelastic Tailoring—Creative Uses of Unusual Materials," AIAA Paper 87-0976-CP, April 1987.

[12]Peloubet, R. P., "YF16 Active Control System/Structural Dynamics Interaction Instability," AIAA Paper 75-823, May 1975.

[13]Felt, L. R., Huttsell, J., Noll, T. E., and Cooley, D. E., "Aeroservoelastic Encounters," *Journal of Aircraft*, Vol. 16, No. 7, July 1979, pp. 477-483.

[14]Brinks, W. H., "F/A-18 Full Scale Development Test," *The Society of Experimental Test Pilots 24th Symposium Proceedings*, Lancaster, CA, Dec. 1980, p. 38.

[15]Swaim, R. L., "Aeroelastic Interactions with Flight Control," AIAA Paper 83-2219, Aug. 1983.

[16]Freyman, R., "Interactions Between an Aircraft Structure and Active Control Systems," *Journal of Guidance, Control and Dynamics*, Vol. 10, No. 5, Sept.-Oct. 1987, pp. 447-452.

[17]Tolson, R. H., and Sobieszczanski-Sobieski, J., "Multidisciplinary Analysis and Synthesis: Needs and Opportunities," AIAA Paper 85-0584.

[18]Sobieszczanski-Sobieski, J., and Haftka, R. T., "Interdisciplinary and Multilevel Optimum Design," *Computer Aided Optimal Design: Structural and Mechanical Systems*, edited by C. A. Mota Soares, Spring-Verlag, Berlin, 1987.

[19]Weisshaar, T. A., Newsom, J. R., Zeiler, T. A., and Gilbert, M. G., "Integrated Structure/Control Design—Present Methodology and Future Opportunities," 1986 Conference of the International Council of the Aeronautical Sciences, London, England, 1986, ICAS Paper 84-4.8.1.

[20]Ashley, H., "On Making Things the Best—The Aeronautical Uses of Optimization," *Journal of Aircraft*, Vol. 19, No. 1, Jan. 1982, pp. 5-28.

[21]Schmit, L. A., "Structural Analysis—Precursor and Catalyst," *Recent Experiences in Multidisciplinary Analysis and Optimization*, NASA CP-2327, Pt. 1, 1984, pp. 1-17.

[22]Schmit, L. A., "Structural Optimization—Some Key Ideas and Insights," *New Directions in Optimum Structural Design*, edited by E. Atrek, R. H. Gallagher, K. M. Ragsdell, O. C. Zienkiewicz, John Wiley and Sons, 1984.

[23]Hornlein, H. R. E. M., "Takeoff in Optimum Structural Design," *Computer Aided Optimal Design: Structural and Mechanical Systems*, edited by C. A. Mota Soares, Springer-Verlag, Berlin, 1987.

[24]Wilkinson, K., Markowitz, J., Lerner, E., George, D., and Batill, S. M. "FASTOP—A Flutter and Strength Optimization Program for Lifting Surface Structures," *Journal of Aircraft*, Vol. 14, No. 6, June 1977.

[25]Markowitz, J., and Isakson, G., "FASTOP3—A Strength, Deflection and Flutter Optimization Program for Metallic and Composite Structures," Air Force Flight Dynamic Lab, Wright Aeronautical Laboratories, Dayton, OH, AFFDL-TR-78-50, May 1978.

[26]Isakson, G., Pardo, H., Lerner, E., and Venkayya, V. B., "ASOP3—A Program for Optimum Structural Design to Satisfy Strength and Deflection Constraints," *Journal of Aircraft*, Vol. 15, No. 7, July 1978, pp. 422-428.

[27]Lerner, E., "The Application of Practical Optimization Techniques in the Preliminary Structural Design of a Forward—Swept Wing," *The Second International Symposium on Aeroelasticity and Structural Dynamics*, Aachen, W. Germany, April 1985, DGLR-Bericht 85-02.

[28]Love, M., and Bohlman, J., "Aeroelastic Tailoring and Integrated Wing Design," *Recent Advances in Multidisciplinary Analysis and Optimization*, NASA CP-3031, Pt. 1, 1989, pp. 431-444.

[29]Lynch, R. W., Rogers, W. A., and Braymen, W. W., "Aeroelastic Tailoring of Advanced Composite Structures for Military Aircraft," AFFDL-TR-76-100, Vol. 1, April 1977.

[30]Triplett, W. E., "Aeroelastic Tailoring Studies in Fighter Aircraft Design," *Journal of Aircraft*, Vol. 17, No. 7, July 1980, pp. 508-513.

[31]Triplett, W. E., "Flutter Optimization in Fighter Aircraft Design," *Recent Experiences in Multidisciplinary Analysis and Optimization*, NASA CP-2327, 1984, pp. 47-63.

[32]Haftka, R. T., "Automated Procedure for Design of Wing Structures to Satisfy Strength and Flutter Requirements," NASA TN D-7264, 1973.

[33]Haftka, R. T., "Optimization of Flexible Wing Structures Subject to Strength and Induced Drag Constraints," *AIAA Journal*, Vol. 15, No. 8, 1977, pp. 1101-1106.

[34]Starnes, J. H., Jr., and Haftka, R. T., "Preliminary Design of Composite Wings for Buckling, Strength and Displacement Constraints," *Journal of Aircraft*, Vol. 16, No. 2, 1979, pp. 564-570.

[35]Haftka, R. T., "Structural Optimization with Aeroelastic Constraints: A Survey of US Applications," *International Journal of Vehicle Design*, Vol. 7, No. 3-4, 1986, pp. 381-392.

[36]Lecina, G., and Petiau, C., "Advances in Optimal Design with Composite Materials," *Computer Aided Optimal Design: Structural and Mechanical Systems*, edited by C. A. Mota Soares, Springer-Verlag, 1987.

[37]Neill, D. J., Johnson, E. H., and Canfield, R., "ASTROS—A Multidisciplinary Automated Structural Design Tool," AIAA Paper 87-0713, April 1987.

[38]Venkayya, V. B., "Recent Developments in Large Scale Structural Optimization," *Recent Advances in Multidisciplinary Analysis and Optimization*, NASA CP-3031, 1989, pp. 1521-1540.

[39]Nissim, E., and Abel, I., "Development and Application of an Optimization Procedure for Flutter Suppression Using the Aerodynamic Energy Concept," NASA TP-1337, Feb. 1978.

[40]Mayne, D. Q., Polak, E., and Sangiovanni-Vincentelli, A., "Computer Aided Design via Optimization," *Proceedings of the IFAC Workshop on Control Applications of Nonlinear Programming*, Denver, CO, June 1979.

[41]Gangsaas, D., Bruce, K. R., Blight, J. D., and Ly, U. L., "Appli-

cation of Modern Synthesis to Aircraft Control: Three Case Studies," *IEEE Transactions on Automatic Control,* Vol. AC-31, No. 11, Nov. 1986, pp. 995–1013.

[42]Gordon, V. C., and Collins, D. J., "Multi-Input Multi-Output Automatic Design Synthesis for Performance and Robustness," *Journal of Guidance, Control and Dynamics,* Vol. 9, No. 3, May–June 1986, pp. 281–287.

[43]Stimler, D. M., and Polak, E., "Nondifferentiable Optimization in Worst Case Control System Design: A Computational Example," *Proceedings of the IEEE Third Symposium on Computer-Aided Control System Design,* Arlington, VA, Sept. 1986, pp. 102–109.

[44]Mukhopadhyay, V., "Stability Robustness Improvement Using Constrained Optimization Techniques," *Journal of Guidance, Control and Dynamics,* Vol. 10, No. 2, March–April 1987, pp. 172–177.

[45]Mukhopadhyay, V., "Digital Robust Control Law Synthesis Using Constrained Optimization," *Journal of Guidance, Navigation and Control,* Vol. 12, No. 2, March–April 1989, pp. 175–181.

[46]Adams, W. M., and Tiffany, S. H., "Application of Optimization Techniques to the Design of a Flutter Suppression Control Law for the DAST ARW-2," *Recent Experiences in Multidisciplinary Analysis and Optimization,* NASA CP-2327, Pt. 1, 1984, pp. 279–295.

[47]Thwaites, B., *Incompressible Aerodynamics,* Oxford Univ. Press, Dover edition, New York, 1987, Chap. 8.

[48]Jones, R. T., "Three Dimensional Wings of Minimum Pressure Drag," *Theory of Optimum Aerodynamic Shapes,* edited by A. Miele, Academic Press, New York, 1965, Chap. 8.

[49]McDonald, J. W., and Stevens, J. R., "Optimized Design of Subsonic Lifting Surfaces," *Journal of Aircraft,* Vol. 7, No. 5, Sept.-Oct. 1970, pp. 442–447.

[50]Hicks, R. M., Murman, E. M., and Vanderplaats, G. N., "An Assessment of Airfoil Design by Numerical Optimization," NASA TM-X-3092, July 1974.

[51]Feifel, W. M., "Optimization and Design of Three-Dimensional Aerodynamic Configurations of Arbitrary Shape by a Vortex Lattice Method," *Vortex Lattice Utilization,* NASA SP-405, 1976, pp. 71–88.

[52]Lamar, J. E., "Minimum Trim Drag Design for Interfering Lifting Surfaces using Vortex-Lattice Methodology," *Vortex Lattice Utilization,* NASA SP-405, 1976, pp. 89–111.

[53]Hicks, R. M., and Henne, P. A., "Wing Design by Numerical Optimization," *Journal of Aircraft,* Vol. 15, July 1978, pp. 407–412.

[54]Vanderplaats, G. N., "Efficient Algorithm for Numerical Airfoil Optimization," *Journal of Aircraft,* Vol. 16, No. 12, Dec. 1979, pp. 842–847.

[55]Rajeswri, B., and Prabhu, K. R., "Optimum Flap Schedules and Minimum Drag Envelopes for Combat Aircraft," *Journal of Aircraft,* Vol. 24, No. 6, June 1987, pp. 412–414.

[56]Gupta, S. C., "GENMAP—Computer Code for Mission Adaptive Profile Generation," *Journal of Aircraft,* Vol. 25, No. 8, Aug. 1988, pp. 766–768.

[57]McGeer, T., "Wing Design for Minimum Drag with Practical Constraints," *Journal of Aircraft,* Vol. 21, No. 11, Nov. 1984, pp. 879–886.

[58]Lan, C. E., "Application of CONMIN to Wing Design Optimization with Vortex Flow Effect," *Recent Experiences in Multidisciplinary Analysis and Optimization,* NASA CP-2327, Pt. 1, 1984, pp. 297–308.

[59]Bodden, D. S., and Junkins, J. L., "Eigenvalue Optimization Algorithms for Structure/Controller Design Iterations," *Journal of Guidance, Control and Dynamics,* Vol. 8, Nov.-Dec. 1985, pp. 697–706.

[60]Lust, R. V., and Schmit, L. A., "Control-Augmented Structural Synthesis," *AIAA Journal,* Vol. 26, No. 1, Jan. 1988, pp. 86–95.

[61]Onoda, J., and Haftka, R. T., "An Approach to Structure/Control Simultaneous Optimization for Large Flexible Spacecraft," *AIAA Journal,* Vol. 25, No. 8, Aug. 1987, pp. 1133–1138.

[62]Haftka, R. T., "Optimum Control of Structures," *Computer Aided Optimal Design: Structural and Mechanical Systems,* edited by C. A. Mota Sores, Springer-Verlag, Berlin Heidelberg, 1987, pp. 381–388.

[63]Khot, N. S., "Minimum Weight and Optimal Control Design of Space Structures," *Computer Aided Optimal Design: Structural and Mechanical Systems,* edited by C. A. Mota Soares, Springer-Verlag, Berlin Heidelberg, 1987.

[64]Gilbert, M. G., and Schmidt, D. K., "Integrated Structure/Control Law Design by Multilevel Optimization," AIAA Paper 89-3470-CP, Aug. 1989.

[65]Junkins, J. L., and Rew, D. W., "Unified Optimization of Structures and Controllers," *Large Space Structures Dynamics and Control,* edited by S. N. Atluri and A. K. Amos, Springer-Verlag, Berlin Heidelberg, 1988.

[66]Thomas, H. L., and Schmit, L. A., "Control Augmented Structural Synthesis with Dynamic Stability Constraints," AIAA Paper 89-1216, April 1989.

[67]Grossman, B., Gurdal, Z., Strauch, G. J., Eppard, W. M., and Haftka, R. T., "Integrated Aerodynamic/Structural Design of a Sailplane Wing," *Journal of Aircraft,* Vol. 25, No. 9, Sept. 1988, pp. 855–860.

[68]Haftka, R. T., Grossman, B., Eppard, W. M., and Kao, P. J., "Efficient Optimization of Integrated Aerodynamic-Structural Design," *Proceedings of the International Conference on Inverse Design Concepts and Optimization in Engineering Sciences—II,* University Park, PA, Oct. 1987.

[69]Haftka, R. T., Grossman, B., Kao, P. J., Polen, D. M., and Sobieszczanski-Sobieski, J., "Integrated Aerodynamic-Structural Design of a Forward-Swept Transport Wing," *Recent Advances in Multidisciplinary Analysis and Optimization,* NASA CP-3031, 1989, pp. 445–463.

[70]Barthelemy, J. F. M., and Bergen, F. D., "Shape Sensitivity Analysis of Wing Static Aeroelastic Characteristics," *Journal of Aircraft,* Vol. 26, No. 8, Aug. 1989, pp. 712–717.

[71]Kapania, R., Bergen, F., and Barthelemy, J., "Shape Sensitivity Analysis of Flutter Response of a Laminated Wing," AIAA Paper 89-1267, April 1989.

[72]Zeiler, T. A., and Weisshaar, T. A., "Integrated Aeroservoelastic Tailoring of Lifting Surfaces," *Journal of Aircraft,* Vol. 25, No. 1, Jan. 1988, pp. 76–83.

[73]Gilbert, M. G., "Sensitivity Method for Integrated Structure/Active Control Law Design," *Sensitivity Analysis in Eingineering,* NASA CP 2457, 1987.

[74]Karpel, M., "Sensitivity Derivatives of Flutter Characteristics and Stability Margins for Aeroservoelastic Design," AIAA Paper 89-3467, Aug. 1989.

[75]Gilbert, M. G., "Results of an Integrated Structure/Control Law Design Sensitivity Analysis," *Recent Advances in Multidisciplinary Analysis and Optimization,* NASA CP-3031, 1989, pp. 727–745.

[76]Gilbert, M. G., Schmidt, D. K., and Weisshaar, T. A., "Quadratic Synthesis of Integrated Active Controls for an Aeroelastic Forward Swept Wing Aircraft," *Journal of Guidance, Control and Dynamics,* Vol. 7, March–April 1984, pp. 190–196.

[77]Sensburg, O., Schmidinger, G., and Fullhas, K., "Integrated Design of Structures," *Journal of Aircraft,* Vol. 26, No. 3, March 1989, pp. 260–270.

[78]Schweger, J., Sensburg, O., and Berns, H., "Aeroelastic Problems and Structural Design of a Tailless CFC Sailplane," *2nd International Symposium on Aeroelasticity and Structural Dynamics,* Aachen, W. Germany, April 1985.

[79]Morris, S., and Kroo, I., "Aircraft Design Optimization with Multidisciplinary Performance Criteria," *Recent Advances in Multidisciplinary Analysis and Optimization,* NASA CP-3031, 1989, pp. 1219–1235.

[80]Morris, S., and Kroo, I., "Aircraft Design Optimization with Multidisciplinary Performance Criteria," AIAA Paper 89-1265, April 1989.

[81]Livne, E., "An Integrated Approach to the Optimum Design of Actively Controlled Composite Wings," *Recent Advances in Multidisciplinary Analysis and Optimization,* NASA CP-3031, 1989, pp. 897–918.

[82]Sharpe, W. E., and Newton, J. B., "Examples of Static Aeroelastic Effects on Present Combat Aircraft Projects," *Static Aeroelastic Effects on High Performance Aircraft,* AGARD CP-403, 1987.

[83]Schmidinger, G., and Sensburg, O., "Static Aeroelastic Considerations in the Definition of Design Loads for Fighter Airplanes," *Static Aeroelastic Effects on High Performance Aircraft,* AGARD CP-403, 1986.

[84]Booker, D., "Aeroelastic Tailoring for Control and Performance—Are Requirements Compatible?" *Combat Aircraft Maneuverability,* AGARD CP-319, 1981.

[85]Weisshaar, T. A., and Nam, C., "Aeroservoelastic Tailoring for Lateral Control Enhancement," *Recent Advances in Multidisciplinary Analysis and Optimization,* NASA CP-3031, 1989, pp. 803–814.

[86]Moynes, J. F., and Gallagher, J. T., "Flight Control System Design for Ride Qualities of Highly Maneuverable Fighter Aircraft," *Guidance and Control Design Considerations for Low Altitude and Terminal Area Flight,* AGARD CP-240.

[87]Swaim, R. L., and Yen, W. Y., "Effects of Dynamic Aeroelasticity on Aircraft Handling Qualities," *Journal of Aircraft,* Vol. 16, No. 9, Sept. 1979, pp. 635–637.

[88]Swaim, R. L., and Poopaka, S., "An Analytical Pilot Rating

Method for Highly Elastic Aircraft," *Journal of Guidance, Control and Dynamics,* Vol. 5, No. 6, Nov.-Dec. 1982, pp. 578-582.

[89]Rimer, M., Chipman, R., and Muniz, B., "Control of a Forward Swept Wing Configuration Dominated by Flight Dynamics/Aeroelastic Interations," *Journal of Guidance, Control and Dynamics,* Vol. 9, No. 1, Jan.-Feb. 1986, pp. 72-79.

[90]Mcruer, D., Johnston, D., and Myers, T., "A Perspective on Superaugmented Flight Control: Advantages and Problems," *Journal of Guidance Control and Dynamics,* Vol. 9, No. 5, Sept.-Oct. 1986, pp. 530-540.

[91]Beaufrere, H., "Limitations of Statically Unstable Aircraft due to the Effects of Sensor Noise, Turbulence, and Structural Dynamics," AIAA Paper 86-2203, Aug. 1986.

[92]Yamamoto, T., "Impact of Aircraft Structural Dynamics on Integrated Control Design," AIAA Paper 83-2216.

[93]Becker, J., Weiss, F., and Sensburg, O., "Compatibility Aspects of Active Control Technologies with Aircraft Structural Design," *Structural Control,* edited by H. H. E. Leipholz, Matrinus Nijhoff Publishers, Boston, 1987.

[94]Sobieszczanski-Sobieski, J., James, B. B., and Dovi, A. R., "Structural Optimization by Multilevel Decomposition," *AIAA Journal,* Vol. 23, Nov. 1985, pp. 1775-1782.

[95]Wrenn, G. A., and Dovi, A. R., "Multilevel Decomposition Approach to the Preliminary Design of a Transport Aircraft Wing," *Journal of Aircraft,* Vol. 25, No. 7, July 1988, pp. 632-638.

[96]Rodden, W. P., Comment on "The Role of Structural and Aerodynamic Damping on the Aeroelastic Behavior of Wings," *Journal of Aircraft,* Vol. 25, No. 3, March 1988, pp. 286-287.

[97]Rodden, W. P., "Comment on General Formulation of the Aeroelastic Divergence of Composite Swept-Forward Wing Structures," *Journal of Aircraft,* Vol. 26, No. 7, July 1989, pp. 694-695.

[98]Livne, E., Schmidt, L. A., and Friedmann, P., *Design Oriented Structural Analysis for Fiber Composite Wings,* UCLA Rept. UCLA-ENG-88-36, Nov. 1988.

[99]Giles, G. L., "Equivalent Plate Analysis of Aircraft Wing Box Structures with General Planform Geometry," *Journal of Aircraft,* Vol. 23, No. 11, Nov. 1986, pp. 859-864.

[100]Giles, G. L., "Further Generalization of an Equivalent Plate Representation for Aircraft Structural Analysis," *Journal of Aircraft,* Vol. 26, No. 1, Jan. 1989, pp. 67-74.

[101]Lottati, I., and Nissim, E., "Three Dimensional Oscillatory Piecewise Continuous Kernel Function Method" (in three parts), *Journal of Aircraft,* Vol. 18, No. 5, May 1981, pp. 346-363.

[102]Lottati, I., and Nissim, E., "Nonplanar, Subsonic, Three Dimensional Oscillatory Piecewise Continuous Kernel Function Method," *Journal of Aircraft,* Vol. 22, No. 12, Dec. 1985, pp. 1043-1048.

[103]Nissim, E., and Lottati, I., "Supersonic Three Dimensional Oscillatory Piecewise Continuous Kernel Function Method," *Journal of Aircraft,* Vol. 20, No. 8, Aug. 1983, pp. 674-681.

[104]Lottati, I., "Induced Drag and Lift of Wing by the Piecewise Continuous Kernel Function Method," *Journal of Aircraft,* Vol. 21, No. 11, Nov. 1984, pp. 833-834.

[105]Landahl, M. T., and Stark, V. J. E., "Numerical Lifting Surface Theory—Problems and Progress," *AIAA Journal,* Vol. 6, No. 11, 1968, pp. 2049-2060.

[106]Rowe, W. S., "Comparison of Analysis Methods used in Lifting Surface Theory," *Computational Methods in Potential Aerodynamics,* edited by L. Morino, Springer-Verlag, Berlin, 1985.

[107]Schmidt, W., "Advanced Numerical Methods for Analysis and Design in Aircraft Aerodynamics," *International Journal of Vehicle Design,* Vol. 7, No. 3/4, 1986, pp. 415-440.

[108]Rogers, W. A., Brayman, W. W., and Shirk, M. H., "Design, Analysis, and Model Tests of an Aeroelastically Tailored Lifting Surface," *Journal of Aircraft,* Vol. 20, No. 3, March 1983, pp. 208-215.

[109]Rodden, W. P., and Giesing, J. P., "Application of Oscillatory Aerodynamic Theory to Estimation of Dynamic Stability Derivatives," *Journal of Aircraft,* Vol. 7, No. 3, May-June 1970, pp. 272-275, (Errata and Addenda in *Journal of Aircraft,* Vol. 21, No. 1, Jan. 1984, pp. 93-94).

[110]Rodden, W. P., and Bellinger, E. D., "Aerodynamic Lag Functions, Divergence, and the British Flutter Method," *Journal of Aircraft,* Vol. 19, No. 7, July 1982, pp. 596-598.

[111]Roger, K. L., "Airplane Math Modeling Methods for Active Control Design," *Structural Aspects of Active Controls,* AGARD CP-228, Aug. 1977, pp. 4-11.

[112]Karpel, M., "Design for Active Flutter Suppression and Gust Alleviation using State Space Aeroelastic Modeling," *Journal of Aircraft,* Vol. 19, No. 3, March 1982, pp. 221-227.

[113]Tiffany, S. H., and Adams, W. M., "Nonlinear Programming Extensions to Rational Function Approximations of Unsteady Aerodynamics," AIAA Paper 87-0854, April 1987.

[114]Tsai, S. W., and Hahn, H. T., *Introduction to Composite Materials,* Technomic, Lancaster, PA, 1980.

[115]Hajela, P., "A Root Locus-Based Flutter Synthesis Procedure," *Journal of Aircraft,* Vol. 20, No. 12, Dec. 1983, pp. 1021-1027.

[116]Haftka, R. T., and Kamat, M. P., *Elements of Structural Optimization,* Martinus Nijhoff Publishers, 1985, Chap. 6.

[117]Bisplinghoff, R. L., and Ashley, H., *Principles of Aeroelasticity,* Dover edition, John Wiley and Sons, New York, 1975.

[118]Taylor, A. S., and Woodcock, D. L., "Mathematical Approaches to the Dynamics of Deformable Aircraft," British Reports and Memoranda, London, No. 3776, June 1971.

[119]Schwanz, R. C., "Consistency in Aircraft Structural and Flight Control Analysis," *Structural Aspects of Active Controls,* AGARD CP-228, 1977.

[120]Rodden, W. P., and Love, J. R., "Equations of Motion of a Quasisteady Flight Vehicle Utilizing Restrained Static Aeroelastic Characteristics," *Journal of Aircraft,* Vol. 22, No. 9, Sept. 1985, pp. 802-809.

[121]Waszak, M. R., and Schmidt, D. K., "Flight Dynamics of Aeroelastic Vehicles," *Journal of Aircraft,* Vol. 25, No. 6, June 1988, pp. 563-571.

[122]Noll, R. B., and Morino, L., "FCAP—A New Tool for the Evaluation of Active Control Technology," AIAA Paper 75-1059, Aug. 1975.

[123]Noll, T., Blair, M., and Cerra, J., "ADAM, An Aeroservoelastic Analysis Method for Analog or Digital Systems," *Journal of Aircraft,* Vol. 23, No. 11, Nov. 1986, pp. 852-858.

[124]Gupta, K. K., Brenner, M. J., and Voelker, L. S., "Integrated Aeroservoelastic Analysis Capability with X-29A Comparisons," *Journal of Aircraft,* Vol. 26, No. 1, Jan. 1989, pp. 84-90.

[125]Peele, E. L., and Adams, W. M., "A Digital Program for Calculating the Interaction between Flexible Structures, Unsteady Aerodynamics and Active Controls," NASA TM-80040, Jan. 1979 (available from NTIS as N81-27115).

[126]Mukhopadhyay, V., Newsom, J. R., and Abel, I., "A Method for Obtaining Reduced Order Control Laws for High Order Systems Using Optimization Techniques," NASA TP-1876, 1981.

[127]Karpel, M., "Time Domain Aeroservoelastic Modeling Using Weighted Unsteady Aerodynamic Forces," *Journal of Guidance, Control and Dynamics,* Vol. 13, No. 1, pp. 30-37.

[128]Pototzki, A. S., Zeiler, T. A., Perry, B., "Time Correlated Gust Loads Using Matched Filter Theory and Random Process Theory—A New Way of Looking at Things," AIAA Paper 89-1374, April 1989.

[129]Campbell, C. W., "Monte Carlo Turbulence Simulation Using Rational Approximations to von Karman Spectra," *AIAA Journal,* Vol. 24, Jan. 1986, pp. 62-66.

[130]Haftka, R. T., and Yates, E. C., Jr., "Repetitive Flutter Calculations in Structural Design," *Journal of Aircraft,* Vol. 13, No. 7, July 1976, pp. 454-461.

[131]Johnson, E. H., "Optimization of Structures Undergoing Harmonic or Stochastic Excitation," Stanford Univ., CA, SUDAAR 501, Aug. 1976.

[132]Hajela, P., and Bach, C. T., "Optimum Structural Sizing for Gust Induced Response," *Journal of Aircraft,* Vol. 26, No. 4, April 1989, pp. 395-397.

[133]Bryson, A. E., and Ho, Y. C., *Applied Optimal Control,* Revised Printing, Hemisphere Publishing Corp., Washington, New York, London, 1975.

[134]Golub, G. H., Nash, S., and Van Loan, C., "A Hessenberg-Schur Method for the Problem $AX + XB = C$," *IEEE Transactions on Automatic Control,* Vol. AC-24, No. 6, Dec. 1979, pp. 909-913.

[135]Moore, R. L., "Aeroservoelastic Stability Analysis of an Airplane with a Control Augmentation System," Ph.D. Thesis, Ohio State Univ., Columbus, OH, 1978 (available from University Microfilms International, No. 7902191).

[136]Droste, C. S., and Walker, J. E., *The General Dynamics Case Study on the F-16 Fly-By-Wire Flight Control System,* AIAA Professional Study Series, New York.